

What can we learn by probing Trans-Planckian physics¹

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In this talk we address the issue of how the observables in our present Universe are affected by processes that may have occurred at superplanckian energies (referred to as the *transplanckian regime*). For example, the origin of the cosmological perturbation spectrum. We model the transplanckian regime by introducing a 1-parameter family of smooth non-linear dispersion relations which modify the frequencies at very short distances. For this family of dispersions, we present the exact solutions and show that the CMBR spectrum is that of a (nearly) black body, and that the adiabatic vacuum is the only choice for the initial conditions. A particular feature of the family of dispersion functions chosen is the production of ultralow frequencies at very high momenta k (for $k > M_P$). Modes with ultralow frequencies equal or less than the current Hubble rate are still frozen today. Therefore, their energy today provides a strong candidate for the dark energy of the Universe.

1 Introduction

Nowadays, quantum field theory is viewed as an effective description of collective degrees of freedom valid below some cutoff scale. If not other, in particle physics the natural cutoff scale is provided by the Planck mass, near which quantum gravity effects will become important. In the definition of an effective field theory, it is implicit that the long wavelength phenomena are decoupled from the small scale processes, such that we do not need to know about the short distance behavior of the underlying fundamental theory in order to study the long distance description. However, there are cases where this long/short distance physics separation breaks down. For example, when studying the origin of the Hawking radiation in Black Hole physics; or when studying the spectrum of primordial cosmological perturbations generated during inflation. In both cases the physical momentum gets blue-shifted back in time, such that the low energy modes evolve from degrees of freedom above the cutoff. Therefore, the effective field theory approach is not viable due to the presence of a strong redshift which mixes the ultraviolet and infrared regimes.

The problem was first raised in Black Hole physics, trying to explain the origin of Hawking radiation. In a series of papers [1, 2, 3], it was demonstrated that the Hawking radiation remains unaffected by modifications of the ultra high energy regime, expressed through the modification of the usual linear dispersion relation at energies larger than a certain ultraviolet scale k_C . In particular, following

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the sonic black hole analogy, Unruh [2] proposed a dispersion relation that goes asymptotically constant,

$$\omega(k) = k_C \tanh^{1/n} [(k/k_C)^n] , \quad (1)$$

whilst Corley and Jacobson [1] adopted the function

$$\omega(k) = k^2(1 - k^2/k_C^2) , \quad (2)$$

which they considered as the lowest order term in the derivative expansion of a generic dispersion relation.

If we think in terms of waves propagating in an inhomogeneous medium, it is reasonable to assume that the dispersion relation for the mode propagation will get modified when the mode start probing the underlying structure of the background; in our case, the transplanckian regime. But we lack a fundamental theory, valid at all energies, able to describe the transition. At the same time, this makes the model building of the transplanckian regime very interesting. The main issue is how much are the known observables affected by the unknown theory. The apparently *ad hoc* modification of the dispersion relation introduced at high energies is constrained by the criterion that its low energy predictions do not conflict the observables. In the case of an expanding Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime, one can ask whether the standard predictions of inflation are or not sensitive to trans-planckian physics. Martin and Brandenberger in Ref. [4] (see also [5, 6]) studied this adopting the above dispersion relations, Eqs. (1) and (2), and found that indeed different dispersion relations lead to different results for the CMBR spectrum. Deviation from the standard scale invariant spectrum were obtained when using Corley and Jacobson's dispersion function, which is not well defined for $k \gg k_C$. They conjecture that the observed power spectrum can always be recovered by using a smooth dispersion relation, which ensures an adiabatic time-evolution of the modes.

In Refs. [4, 5] the authors have also demonstrated that the problem of calculating the spectrum of perturbations with a time-dependent dispersive frequency can be reduced to the familiar topic of particle creation on a time-dependent background [7]. In this talk we adopt their method in studying the trans-planckian problem, but we introduce a new family of dispersion relations to model that regime [8]. This class of functions has the following features: it is smooth, nearly linear for energies less than the Planck scale, reaches a maximum, and attenuates to zero at ultrahigh momenta thereby producing ultralow frequencies at very short distances (see fig. (1)). This choice of functions is motivated by superstring duality [9] (which applies at transplanckian energies). Our family of dispersion relations exhibits *dual* behavior, i.e., appearance of ultra-low mode frequencies both at low and high momenta.

Below we present the exact solutions to the mode equation, and the resulting CMBR spectrum. The major contribution to the CMBR spectrum comes from the long wavelength modes when they re-enter the horizon. The spectrum is nearly insensitive to the very short wavelength modes inside the Hubble horizon.

Therefore, by taking the frequency dispersion relations to be the general class of Epstein functions [10], we check and lend strong support to the conjecture made in Ref. [4].

On the other hand, the family of dispersion chosen present the distinctive feature of having a “tail” of modes with ultralow frequencies, less or equal to the current Hubble constant H_0 . It follows that the *tail* modes are still currently frozen. They provide a unique candidate for the dark energy of the universe (see Ref. [8]).

2 The Model and CMBR spectrum

The initial power spectrum of the metric perturbations can be computed once we solve the time-dependent equations for the scalar and tensor sector. The mode equations for both sectors reduce [11] to a Klein-Gordon equation of the form

$$\mu_n'' + \left[n^2 - \frac{a''}{a} \right] \mu_n = 0, \quad (3)$$

where the prime denotes derivative with respect to conformal time. Therefore, studying perturbations in a FLRW background is equivalent to solving the mode equations for a scalar field μ related (through Bardeen variables [11]) to the perturbation field in the expanding background. The dynamics of the scale factor is determined by the evolution of the background inflaton field ϕ , and the Friedmann equation. We will consider the class of inflationary scenarios that has a power law solution for the scale factor $a(\eta)$ in conformal time, $a(\eta) = |\eta_c/\eta|^\beta$, with $\beta \geq 1$.

Eq. (3) represents a linear dispersion relation for the frequency ω , $\omega^2 = k^2 = n^2/a^2$. This dispersion relation holds for values of momentum smaller than the Planck scale. There is no reason to believe that it remains linear at ultra-high energies larger than M_P . However, we should stress that any modeling of Planck scale physics even by analogy with already familiar systems is pure speculation. We lack the fundamental theory that may naturally motivate or reproduce such dispersive behavior.

In what follows, we will replace the linear relation with a nonlinear dispersion relation $\omega(k) = F(k)$. Therefore, in Eq. (3), n^2 should be replaced by $n_{eff}^2 = a(\eta)^2 F(k)^2 = a(\eta)^2 F[n/a(\eta)]^2$. For future reference, we also define the generalised comoving frequency as $\Omega_n^2 = n_{eff}^2 - a''/a$. We consider the following Epstein function [10] for the dispersion relation:

$$\omega^2(k) = F^2(k) = k^2 \left(\frac{\epsilon_1}{1+e^x} + \frac{\epsilon_2 e^x}{1+e^x} + \frac{\epsilon_3 e^x}{(1+e^x)^2} \right), \quad (4)$$

$$n_{eff}^2 = a^2(\eta) F^2(n, \eta) = n^2 \left(\frac{\epsilon_1}{1+e^x} + \frac{\epsilon_2 e^x}{1+e^x} + \frac{\epsilon_3 e^x}{(1+e^x)^2} \right), \quad (5)$$

where $x = (k/k_C)^{1/\beta} = A|\eta|$, with $A = (1/|\eta_c|)(n/k_C)^{1/\beta}$. This is the most general expression for this family of functions. For our purposes, we will constrain some

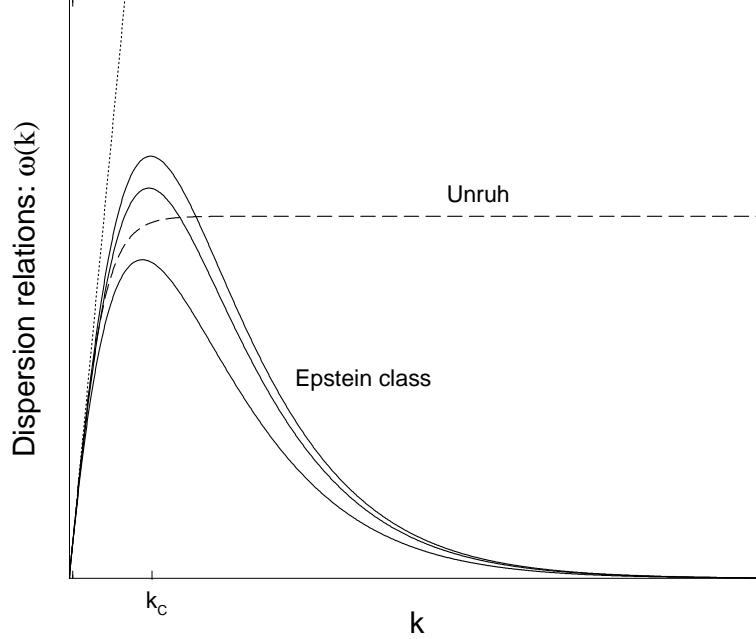


Figure 1: Shown is our family of dispersion relations, for $\beta = 1$ and representative values of ϵ_1 (solid lines). We have also included the Unruh's dispersion relation (dashed line) and the linear one (dotted line) for comparison.

of the parameters of the Epstein family in order to satisfy the features required for the dispersion relation as follows. First, in order to have ultralow frequencies for very high momenta, we demand that the dispersion function goes asymptotically to zero. That fixes $\epsilon_2 = 0$. And the condition of a nearly linear dispersion relation for $k < k_C$ requires that $2\epsilon_1 + \epsilon_3 = 4$. Still we will have a whole family of functions parametrised by the constant ϵ_1 , as can be seen in Fig. 1.

Eq. (3) with the new dispersion function Eq. (5) is exactly solvable in terms of hypergeometric functions [10]. This is a well studied case in the context of particle creation in a curved background [7]. The contribution from a''/a is going to be negligible at early times ($\eta \rightarrow -\infty$); at late times, it can be absorbed in the dispersion relation Eq. (5) redefining the constants ϵ_i .

The correct initial condition is the vacuum state solution that minimizes the energy [12]. The choice of the correct vacuum state is important, since most of the contribution to the spectrum of perturbations comes from long-wavelength modes. They are produced at early stages of inflation, being very sensitive to the initial conditions. When $\epsilon_2 \neq 0$, the vacuum state behaves as a plane wave in the asymptotic limit $\eta \rightarrow -\infty$, with $\Omega_n^{(in)} \rightarrow \sqrt{\epsilon_2}n$. However, when $\epsilon_2 = 0$ as in our case, the correct behavior of the mode function in the remote past is given by the solution of its evolution equation in the limit $\eta \rightarrow -\infty$. The exact solution which matches this asymptotic behavior is then given by:

$$\mu^{(in)}(\eta) = C^{in} \left(\frac{1+u}{u} \right)^d {}_2F_1 \left[\frac{1}{2} + d + b, \frac{1}{2} + d - b, 1 + 2d, \frac{1+u}{u} \right], \quad (6)$$

where $u = \exp(A|\eta|)$, C^{in} is a normalization constant, and

$$b = i\tilde{b} = i\sqrt{\hat{\epsilon}_1}, \quad d = i\tilde{d} = \sqrt{\frac{1}{4} + \hat{\epsilon}_3}, \quad (7)$$

where $\hat{\epsilon}_i = (k_C|\eta_c|)^2(n/k_C)^{2(1-1/\beta)}\epsilon_i$. At late times, the solution becomes a squeezed state by mixing of positive and negative frequencies:

$$\mu_n \rightarrow_{\eta \rightarrow +\infty} \frac{\alpha_n}{\sqrt{2\Omega_n^{out}}} e^{-i\Omega_n^{out}\eta} + \frac{\beta_n}{\sqrt{2\Omega_n^{out}}} e^{+i\Omega_n^{out}\eta}, \quad (8)$$

where α_n and β_n are the Bogoliubov coefficients, and $\Omega_n^{out} \simeq \sqrt{\epsilon_1}n$; $|\beta_n|^2$ gives the particle creation number per mode n . Using the linear transformation properties of hypergeometric functions [13], we find that

$$\left| \frac{\beta_n}{\alpha_n} \right| = e^{-2\pi\tilde{b}} \left| \frac{\cosh \pi(\tilde{d} + \tilde{b})}{\cosh \pi(\tilde{d} - \tilde{b})} \right|. \quad (9)$$

It is clear from Eq. (9) that the spectrum of created particles is nearly thermal to high accuracy²: $|\beta_n|^2 \simeq e^{-4\pi\tilde{b}}$. Thus, we can immediately conclude that the CMBR spectrum is that of a (nearly) black body spectrum. This means that the spectrum is (nearly) scale invariant, i.e., the spectral index is $n_s \simeq 1$. This is consistent with previous results obtained in the literature [4, 5, 6], when using a smooth dispersion relation and the correct choice of the initial vacuum state, as discussed above. In Refs. [4] and [5], dispersion relations that were originally applied to black hole physics [1, 2] were used in the context of cosmology. New models of dispersion relations were proposed by the authors of Refs. [6]. Our proposal for a 1-parameter class of models has a significantly different feature from the above, namely: the appearance of ultra-low frequency modes in the transplanckian regime. The implications of such a behavior for high momenta on the production of dark energy are discussed in Ref. [8].

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² We mention that we have neglected the backreaction effects during the calculation. However, this is consistent with the result obtained of a small particle number per mode, in the high momentum regime ($k \gg M_P$) and a very small energy contained in these modes. Because of these results, we do not have the problems mentioned in Ref. [14] when discussing trans-planckian physics. For the energy see Ref. [8].

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